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Quantitative analysis of illusory movement: spatial filtering and line localization in the human visual system

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ABSTRACT

30 A narrow bar or line (width around 1 arcminute) between two fields of which the luminances are sinusoidally and in counterphase modulated in time appears to make an oscillatory movement. It is possible to annihilate this illusory movement with a real movement and thus to analyze this phenomenon quantitatively. Confirming previous studies, the amount of illusory movement (amplitude typically 10 arcsecond) was proportional to the modulation depth of the fields and inversely
35 proportional to the line width and the line contrast. The amount of illusory movement increased with defocus, a lower mean luminance, and eccentricity. The experimental results could be explained by a model that includes a linear low-pass spatial filter. For a Gaussian spatial filter, the standard deviation as derived from the experimental results was 1.1 (1.0-1.3) arcminute (median with range of 4
40 observers) for well-focused, photopic, foveal viewing. We explored various criteria for line localization in the model (extremes and zero crossings of Gaussian derivatives).

Keywords: spatial vision, resolution, apparent movement, hyperacuity, vernier acuity

INTRODUCTION

A narrow bar or line positioned between two fields of which the luminances are modulated in counterphase in time, appears to make an oscillatory movement at a frequency which equals the modulation frequency. This illusory movement was reported half a century ago by Veringa (1961) as a side-issue of his experiments with modulated light, and systematically investigated by Gregory and Heard (1983) and Jansonius and Kuiper (1989). Its size is typically less than an arcminute. Illusory movement should be distinguished from phenomena such as short-range apparent motion, of which the range extends to approximately 15 arcminute in the fovea (Braddick, 1974; Allik & Pulver, 1995) and increases with eccentricity (Baker & Braddick, 1985), and long-range apparent motion, of which the range extends over many degrees (Zeeman & Roelofs, 1953; Braddick, 1980; Larsen, Farrell, & Bundesen, 1983).

Illusory movement can be explained by a model that includes low-pass spatial filtering in the visual system (Anstis & Rogers, 1975; Mastebroek & Zaagman, 1988). Figure 1 illustrates the concept. The image of a dark line in between two fields with different luminances appears to be shifted towards the field with the lowest luminance, if low-pass filtered. The illusory movement can be compensated (nulled) with a real movement, and this enables a quantitative analysis of the spatial filtering.

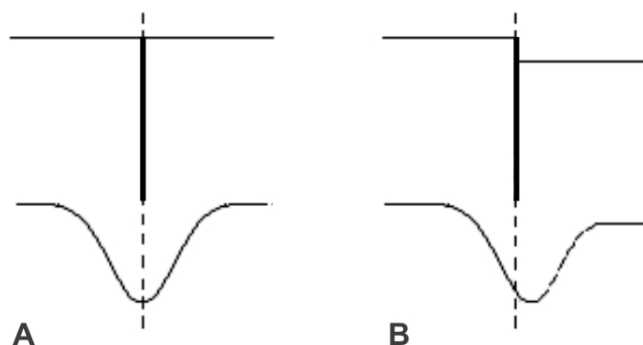


Figure 1 Low-pass filtered image of a dark line in between two fields with different luminances appears to be shifted towards the field with the lowest luminance (**A**: equal luminances; **B**: different luminances).

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Marr and Hildreth (1980) proposed a theory of edge detection, with the location of an edge represented by the zero-crossing of the convolution of the edge with the second derivative (Laplacian) of a Gaussian spatial filter. This approach gives an unambiguous estimate of the location of an edge, but not of a line. In the case of a line, two zero-crossings will occur and neither of them is aligned with
75 the line. The zero-crossing of the convolution of a line with the first derivative of a Gaussian spatial filter, on the other hand, representing the location of the extreme (maximum in the case of a bright line; minimum in the case of a dark line), gives an unambiguous estimate of the location of the line (Watt, 1988). Others have argued that the first derivative is of little importance and that the third (and fourth) derivative(s) supplement the second derivative (Koenderink & van Doorn, 1987), or that the
80 features that are used in the localization depend on the spatial characteristics of the stimulus and on the actual task (Toet, Smit, Nienhuis, & Koenderink, 1988).

The first aim of this study was to reproduce the phenomenon and to study quantitatively the influence of the modulation depth of the fields, the line width, the contrast of the line, optical blur (defocus), the
85 mean luminance of the fields, and eccentricity on illusory movement. The second aim was to retrieve the characteristics of the spatial filter that could explain the experimental findings. For this purpose, we modeled the effect of spatial filtering of the stimulus and we compared the calculated illusory movement (shift of line location) with the experimental results. This was done for several criteria for line localization (extremes and zero crossings of Gaussian derivatives).

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METHODS

Experiments

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Subjects

Four healthy subjects participated in this study (all male; TJ 26, LS 22, NJ 46, and FJ 27 years of age). None of the subjects had a history of eye disease except for refractive error. Most experiments were performed monocularly; the dominant eye was used. The median (range) spherical-equivalent refractive error of the dominant eyes of the subjects was -0.50 (-2.25 to +0.75) D (TJ -2.25 D; LS +0.75 D; NJ -1.5 D; FJ +0.50 D). Spherical refractive error and astigmatism were corrected within ± 0.25 D for the viewing distance. With correction, all eyes had a visual acuity of at least 1.0 (6/6). The experiments were carried out in a sparsely illuminated room. No cycloplegia, mydriasis or artificial pupil was used. The median (range) pupil diameter during the experiments was 5.5 (5.0 to 6.0) mm (TJ 5 mm; LS 6 mm; NJ 5 mm; FJ 6 mm). All participants provided written informed consent. This study was a pilot experiment for a study in glaucoma patients and healthy controls, approved by the ethics board of the University Medical Center Groningen. The study followed the tenets of the Declaration of Helsinki.

Setup, stimulus and data acquisition

The stimulus was generated on a CRT monitor (Philips 109B; Philips, Eindhoven, the Netherlands). The frame rate was 85 Hz; the mean luminance of the screen was 63 cd/m² as measured with a Minolta luminance meter with built-in photometric filter (LS-110; Minolta Camera Co. Ltd., Japan). The frame rate is far beyond the critical fusion frequency, which is about 40 Hz at this luminance (Hecht & Verrijp, 1933; de Lange, 1954). The stimulus was generated and the data were collected using Matlab (version 7.10.0 R2010a; Mathworks, Nattick, MA, USA) in combination with the Psychophysics Toolbox (PTB-3; Brainard, 1997; Pelli, 1997). The distance between subject and screen was 6 m. The screen size was about 4 degree (0.4 m at 6 m); the stimulus size was limited by a mask to 1 by 1 degree. The luminance of the area outside the mask was equal to the mean luminance of the stimulus.

Figure 2 depicts the stimulus. The stimulus was a narrow, vertical bar (line) positioned between two fields. The fields had equal mean luminance, and their luminances were sinusoidally and in counterphase modulated in time. The luminance of the screen $L(x,t)$ is given by:

$$L(x,t) = \begin{cases} L_m (1 - m \sin \omega t) & \text{for } x < -\alpha + a \sin \omega t \\ L_L & \text{for } -\alpha + a \sin \omega t \leq x \leq \alpha + a \sin \omega t \\ L_m (1 + m \sin \omega t) & \text{for } x > \alpha + a \sin \omega t, \end{cases} \quad (1)$$

where x is the horizontal position on the screen, t the time, L_m the mean luminance, m the modulation depth, $\omega/2\pi$ the modulation frequency, 2α the line width, a the amplitude of the real movement used to compensate the illusory movement, and L_L the luminance of the line. Line contrast was defined as Weber contrast: $(L_L - L_m)/L_m$.

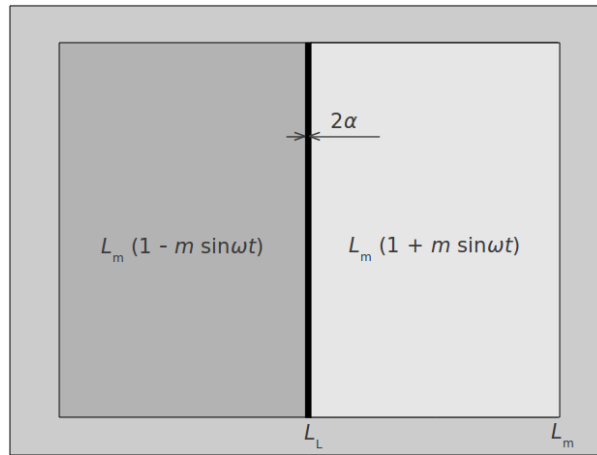


Figure 2 Outline of the stimulus. A narrow, vertical line (line width 2α ; line luminance L_L) is positioned between two fields. The fields have equal mean luminance and their luminances are sinusoidally and in counterphase modulated in time (modulation depth m ; modulation frequency $\omega/2\pi$). The luminance of the surrounding equals the mean luminance of the fields (L_m).

At the testing distance of 6 m, a pixel has a typical width of 0.1 arcminute. This width is small compared to the width of the optical line spread function (typically 1 arcminute; Campbell &

150 Gubbisch, 1966) and the line width (typically 1 arcminute; see below) but not compared to a , the amplitude of the real movement. By allowing the border pixels on both sides of the line to have an intermediate luminance, between the luminance of the line and the luminance of the adjacent field, the spatial position of the energy distribution of the line could be adjusted with a much higher resolution than the resolution of the pixel grid. This relies on spatial summation. Care was taken to keep the
155 integrated energy distribution of the line fixed for a given line width and line luminance. A separately generated look-up table guaranteed a linear relationship between the displayed and internally represented luminance. For one of the subjects (NJ), the experiments were also performed on an analogous setup that did not suffer from the pixel size limitation (Jansonius & Kuiper, 1989) and with another setup with an LCD monitor (Samsung SyncMaster 2243WM). The latter setup was using
160 Octave (version 3.2.4; www.gnu.org/software/octave/) for Linux (Ubuntu 10.10), again in combination with the Psychophysics Toolbox (PTB-3; Brainard, 1997; Pelli, 1997). Results between the three setups were in perfect agreement.

We measured the illusory movement by compensating it with a real movement, using a
165 psychophysical method modified from the semi-automatic audiometer as described by von Békésy (1967). During an experiment, the amplitude a of the real movement of the line changes (increases or decreases) at a constant speed of 0.8 arcsecond per second (Jansonius, 1988). At the beginning of an experiment, a is set at 10 arcminute, a typical value for which no movement is observed for the default stimulus parameters (see below). Initially, a increases, until movement is observed. At this stage, the
170 real movement overcompensates the illusory movement. Now, the subject has to press a button to initiate a decrease of a . As soon as no movement is observed anymore, the button is to be released, and so on. After 12 reversals, a is made so small that the real movement undercompensates the illusory movement and another 12 reversals are recorded. Finally, to compensate for any drift (change in subjective threshold criterion during the experiment), another 12 reversals are recorded for the initial
175 situation. Figure 3 illustrates the course of an experiment.

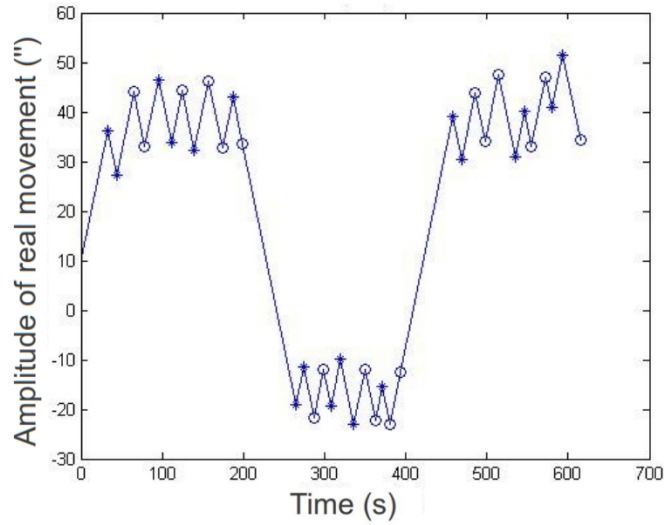


Figure 3 Course of an experiment. Initially, the amplitude of the real movement a increases, until movement is observed. At this stage, the real movement overcompensates the illusory movement. Button press now initiates a decrease of a . As soon as no movement is observed anymore, the button is released. After 12 reversals, the threshold for undercompensation of the illusory movement is determined in 12 reversals, followed by another determination of the overcompensation threshold. The first two reversals of each series as well as the two extremes of both polarities were discarded from the analysis (indicated with an asterisk).

Stimulus parameters

The default stimulus parameters were a modulation depth $m = 0.08$, a line width $2\alpha = 1.2$ arcminute, and a line luminance $L_L = 0$ cd/m² (line contrast $(L_L - L_m)/L_m = -1$). The other default settings were a mean luminance of the fields $L_m = 63$ cd/m² (photopic vision), optimal refractive correction for the viewing distance, and foveal fixation. Experiments were also performed with modulation depths of 0.04 and 0.16, line widths of 0.6 and 2.4 arcminute, and line luminances of 32 and 84 cd/m² (line contrasts of -0.50 and +0.33). Furthermore, we explored the effects of defocus (+1 D blur added to the optimal refractive correction for the viewing distance), mean luminance (by using a neutral density filter with 2% transmission), and eccentricity (by fixating at a distance of 0.5, 1, and 2 degree from the

205 line). All experiments were performed at a modulation frequency of 2.5 Hz (Jansonius, 1988). Most experiments were performed monocularly, with the dominant eye (see above). To avoid accommodation as much as possible, we performed the experiments with defocus and decreased mean luminance binocularly. The default experiment was performed both monocularly and binocularly.

210 *Data analysis*

We discarded the first two reversals of each series as well as the two extremes of both polarities (Nio et al., 2005; indicated with an asterisk in Fig. 3). We averaged the remaining six values (indicated with dots in Fig. 3) to calculate a_1 , a_2 , and a_3 . We defined the amplitude of compensation a_c as:

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$$a_c = (0.5(a_1 + a_3) + a_2) / 2 \quad (2)$$

and the difference threshold Δa between the illusory movement and the real movement as:

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$$\Delta a = |0.5(a_1 + a_3) - a_2| / 2. \quad (3)$$

Results are presented as median values with range. Significance was tested with the paired t-test and two-way analysis of variance.

225 **Model calculations**

As mentioned in the Introduction section, illusory movement has been attributed to low-pass spatial filtering in the visual system. We modeled this filtering by calculating the convolution $g(x,t)$ of the stimulus $L(x,t)$ with a Gaussian spatial filter $h(x)$ with standard deviation σ :

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$$g(x, t) = L(x, t) * h(x) \quad (4)$$

and the convolutions of the stimulus with the first, second, and third derivatives of $h(x)$. The latter convolutions are identical to the first, second, and third derivatives of $g(x, t)$ with respect to x (denoted as $g'(x, t)$, $g''(x, t)$, and $g'''(x, t)$, respectively) because of a general property of convolution:

$$(f * h)' = f * h', \quad (5)$$

where f is a locally integrable function and h a differentiable function. We studied three different criteria for line localization. For these criteria, the location of the line in the visual system is:

(1) where $g(x, t)$ has (for a dark line) a minimum, that is, at the zero crossing of $g'(x, t)$ (Watt, 1988)

(2) at the average position of the two zero crossings of $g''(x, t)$ (following Marr & Hildreth, 1980; feature *iii* in Toet et al., 1988)

(3) where $g''(x, t)$ has (for a dark line) a minimum, that is, at the (central) zero crossing of $g'''(x, t)$

(following Koenderink & van Doorn, 1987; feature *ii* in Toet et al., 1988)

All three criteria refer to specific locations (extremes or zero-crossings) in the luminance profile or its spatial derivatives; it was previously shown that human vision does not use the energy profile to localize features (Georgeson & Freeman, 1997; Hesse & Georgeson, 2005). We modeled luminance linearly (Anstis, 1986; see Discussion section).

Figure 4 shows the stimulus $L(x, t)$ and the convolution $g(x, t)$ of the stimulus with the Gaussian $h(x)$

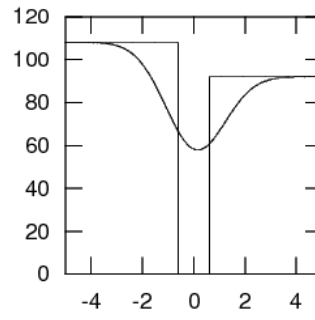
(upper part), and the corresponding convolutions of the stimulus with the first, second, and third

derivatives of $h(x)$, that is, $g'(x, t)$, $g''(x, t)$, and $g'''(x, t)$ (lower three parts). In this figure, standard

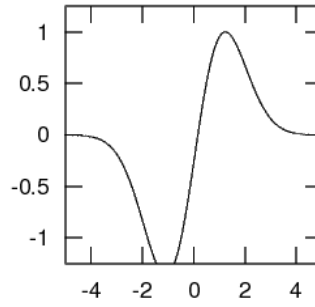
deviation σ of $h(x)$ was 1.1 arcminute. All model calculations were performed using Octave (version

3.2.4; www.gnu.org/software/octave/) for Linux (Ubuntu 10.10).

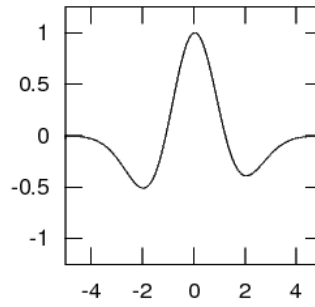
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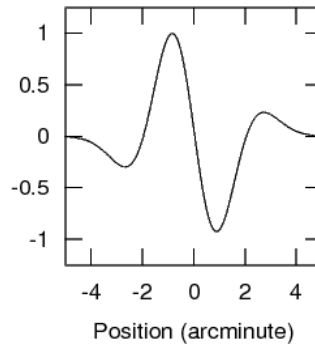


Figure 4 Stimulus $L(x,t)$ and the convolution $g(x,t)$ of the stimulus with a Gaussian $h(x)$ (upper part), and the corresponding convolutions of the stimulus with the first, second, and third derivatives of $h(x)$ ($g'(x,t)$, $g''(x,t)$, and $g'''(x,t)$; lower three parts). Standard deviation σ of $h(x)$ was 1.1 arcminute.

RESULTS

Experiments

For the default experiment (modulation depth $m = 0.08$, line width $2\alpha = 1.2$ arcminute, line luminance $L_L = 0$ cd/m²), the median (range) amplitude of compensation was 11.3 (9.4 to 13.2) arcsecond for monocular viewing and 10.2 (9.3-13.0) arcsecond for binocular viewing. Monocular and binocular amplitude of compensation did not differ significantly (paired t-test; $t(3)=0.77$, $p=.50$). Figure 5 shows that the amplitude of compensation was linearly related to m (Fig. 5A), the inverse of 2α (Fig. 5B), and the inverse of line contrast (Fig. 5C). Table 1 presents the effects of defocus and of reducing the mean luminance of the stimulus. The repeated measurements were significantly different (two-way analysis of variance; $F(2,6)=12.6$, $p=.007$). A blur of +1 D caused a significant increase in amplitude of compensation (mean increase 15.3 arcsecond; $p=.013$; post-hoc analysis with Scheffé procedure). Reducing the mean luminance from 63 to 1.3 cd/m² resulted in a significant increase in amplitude of compensation (mean increase 14.6 arcsecond; $p=.016$). Figure 6 presents the effect of eccentricity. The amplitude of compensation increased with eccentricity (two-way analysis of variance; $F(3,9)=45.5$, $p<.001$) and this increase started already at 0.5 degree ($p=.037$; post-hoc analysis with Scheffé procedure for 0 versus 0.5 degree; mean increase 8.1 arcsecond).

Table 1 Effects of defocus (+1 D spherical blur) and reduced mean luminance (from 63 to 1.3 cd/m²) on the amplitude of compensation (in arcsecond).

	TJ	LS	NJ	FJ	Median
Default	10	9	13	10	10
Defocus	20	21	36	27	24
Reduced mean luminance	29	17	25	30	27

The median (range) difference threshold Δa between the illusory movement and the real movement was 25 (15-30) arcsecond for the default experiment, at the applied modulation frequency of 2.5 Hz.

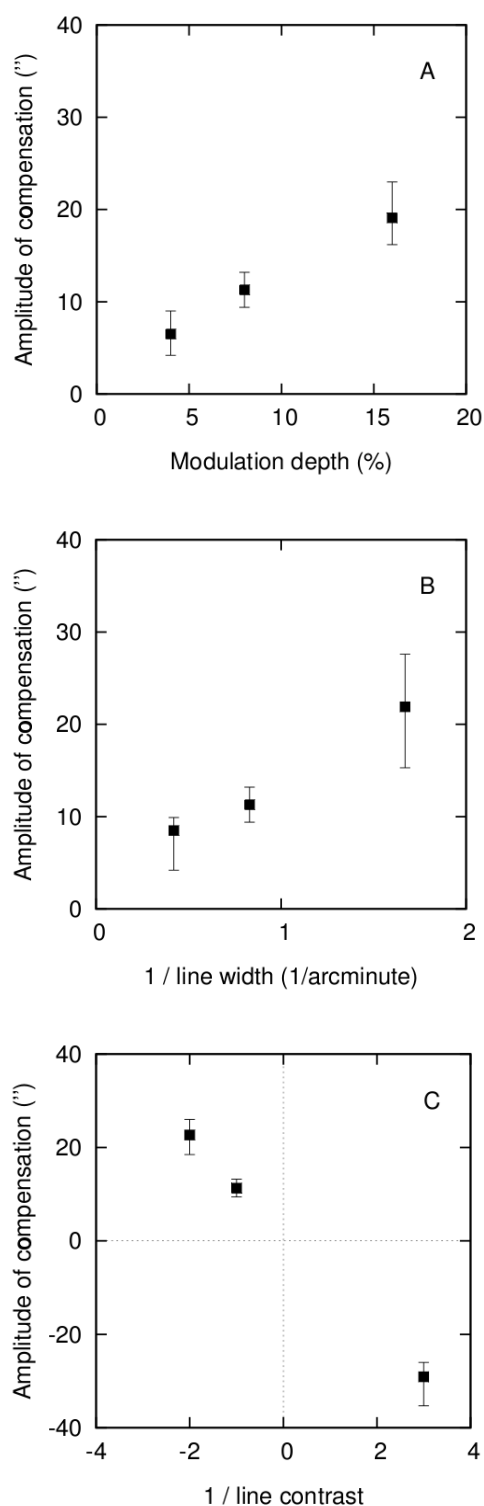


Figure 5 Measured amplitude of compensation (median with range) as a function of modulation depth (**A**), inverse of line width (**B**), and inverse of line contrast (**C**).

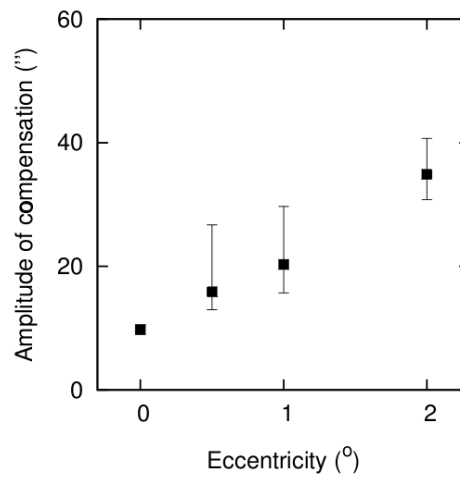
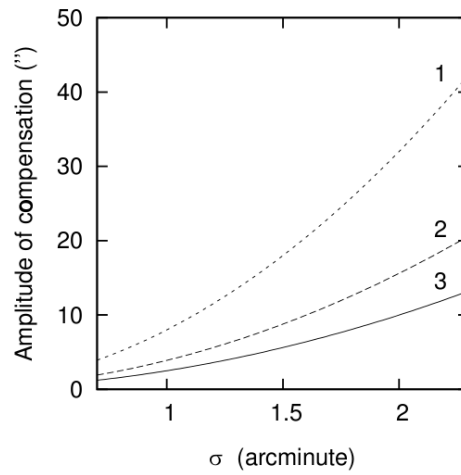


Figure 6 Measured amplitude of compensation (median with range) as a function of eccentricity.

Model calculations

All three criteria for line localization yielded a model that was able to explain illusory movement, that is, the movement (shift) occurred and was in the experimentally observed direction. Figure 7 presents, for the three criteria, the calculated amplitude of compensation as a function of σ for the default experiment (modulation depth $m = 0.08$, line width $2\alpha = 1.2$ arcminute, line luminance $L_L = 0$ cd/m²). As can be seen in this figure, different σ 's were needed for the three criteria to obtain a quantitative agreement between the experimental results and the model calculations. For the experimentally found amplitude of compensation of 10 (9-13) arcsecond, the model calculations predicted σ values of 1.1 (1.0 to 1.3), 1.6 (1.5 to 1.8), and 2.0 (1.9 to 2.2) arcminute for the first, second, and third criterion, respectively. With these σ values, the model calculations were quantitatively in agreement with the experimental results. Table 2 shows the model calculations; all values were within the range of the experimental findings (error bars in Fig. 5). The differences between the three criteria were small compared to the range of the experimental findings.

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Figure 7 Calculated amplitude of compensation as a function of the standard deviation σ of the Gaussian spatial filter $h(x)$ for the three criteria for line position as mentioned in the text. Default parameters (modulation depth $m = 0.08$, line width $2\alpha = 1.2$ arcminute, line luminance $L_L = 0$ cd/m²).

Table 2 Calculated amplitude of compensation (in arcsecond) as a function of modulation depth, line width, and line contrast for the three different criteria for line location

	Criterion 1 with $\sigma = 1.1'$	Criterion 2 with $\sigma = 1.6'$	Criterion 3 with $\sigma = 2.0'$
$m = 0.04$	5.0	5.0	5.0
0.08	10.0	10.0	10.0
0.16	19.5	19.8	20.0
$2\alpha = 0.6'$	19.5	20.3	20.5
1.2'	10.0	10.0	10.0
2.4'	5.0	4.3	4.0
$(L_L - L_m) / L_m = -1.0$	10.0	10.0	10.0
-0.5	19.5	19.8	20.0
+0.33	-29.5	-29.8	-29.0

375 For the first criterion, the relationship between the amplitude of compensation, σ , modulation depth, line width, and line contrast can be derived analytically as well (see Appendix):

$$a_c = \frac{-2 m \sigma^2}{2 \alpha (L_L / L_m - 1)}. \quad (6)$$

380 Equation (6) shows that the amplitude of compensation is proportional to the modulation depth, σ^2 , the inverse of line width, and the inverse of line contrast. With $m = 0.08$, $\sigma = 1.1$ arcminute, $2\alpha = 1.2$ arcminute, and $(L_L / L_m - 1) = -1$, Eq. (6) yields $a_c = 0.16$ arcminute (10 arcsecond). All this is in perfect agreement with the experimental findings (Fig. 5) and model calculations (Fig. 7; Table 2).

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DISCUSSION

We were able to reproduce the existence of illusory movement and the influence of modulation depth, line width, and line luminance on illusory movement: it increases linearly with modulation depth, the
390 inverse of line width, and the inverse of line contrast. Our model calculations confirmed that low-pass spatial filtering provides a useful model for illusory movement. In agreement with this, the phenomenon was more pronounced with defocus, low mean luminance, and eccentric viewing.

As mentioned in the Introduction section, the role of low-pass spatial filtering in illusory movement
395 has been described before (Anstis & Rogers, 1975; Mastebroek & Zaagman, 1988). We systematically compared criteria for line localization and described the spatial filter quantitatively. A model comprising low-pass spatial filtering with a simple Gaussian $h(x)$ with $\sigma = 1.1$ arcminute (Criterion 1) was already able to explain the experimental results adequately. A Gaussian with $\sigma = 1.1$ arcminute has a full width at half height (FWHH) of 2.6 arcminute. Interestingly, this FWHH is in very good

400 agreement with the FWHH of psychophysically determined line spread functions (LSFs) and point
spread functions (PSFs) at a similar luminance (van Meeteren & Vos, 1972; Blommaert & Roufs,
1981). At a sufficiently high luminance, the LSF and PSF have negative side bands, yielding a
sensitivity profile that resembles $h''(x)$. For $h''(x)$, we found $\sigma = 1.6$ arcminute for Criterion 2 and $\sigma =$
2.0 arcminute for Criterion 3. The FWHH of $h''(x)$ for these σ values is essentially equal to the FWHH
405 of $h(x)$ for $\sigma = 1.1$ arcminute (for a given σ , $h''(x)$ has a narrower main lobe than $h(x)$). With that, and
with the results as displayed in Table 2, our experimental findings and model calculations do not allow
for discriminating between the various criteria for line localization. However, our results do indicate
that a model more complex than low-pass spatial filtering in the early visual system is not needed.
Following the principle of parsimony (Occam's razor), such a model might thus be the preferred model
410 to explain illusory movement.

The negative side bands in the LSF and PSF are related to the occurrence of a peak in the contrast
sensitivity function and are presumed to reflect lateral inhibition (see, for example, Robson, 1966;
Westheimer, 1967; van Nes & Bouman, 1967). In agreement with the presence of lateral inhibition,
415 we showed previously a 180 degree phase shift of the illusory movement when using a modified
stimulus with the modulated fields replaced by modulated stripes positioned at various distances from
the line. The phase shift occurred when the distance between the modulated stripes and the line was
approximately 5 arcminute (Jansonius & Kuiper, 1989), a value in good agreement with the results of
Westheimer (1967), van Meeteren and Vos (1972), and Kulikowski and King-Smith (1973).

420 In our model calculations, we modeled a shift (static displacement) rather than a movement. This
seems to be a reasonable approach, especially because we measured the amount of illusory movement
by nulling with a real movement, yielding an apparently static stimulus at the time point of the actual
measurement. Gregory and Heard (1983) measured both illusory movement and illusory displacement
425 and found some differences. This suggests that the assumption underlying our modeling might not be
entirely correct. However, Gregory and Heard used a different psychophysical method and their

results are difficult to interpret due to the geometry of their stimulus. Their stimulus consisted of two rectangles aligned vertically, with dark and light lines along the vertical borders. There was 0.5 degree gap between the rectangles and each rectangle was 1.5 degree in height, yielding a total stimulus height of 3.5 degree. Illusory displacement was quantified by nulling a vernier misalignment. With a gap of 0.5 degree, this is foveal task. Illusory movement was quantified by matching with true movements of a pair of oscillating rectangular line figures that were displayed next to the stimulus. Here, given the size of the total stimulus, no unambiguous matching is to be expected (Fig. 6). Indeed, we had intentionally confined the length of our line to 1 degree because of the occurrence of ambiguity with longer lines. In a supplementary experiment, we found no relationship between the amplitude of compensation and the modulation frequency (1.25 to 5 Hz; outside this range, the discrimination between movement and no movement is very difficult).

We modeled luminance linearly. Georgeson and Freeman (1997) studied the perceived location of bars and edges with model calculations and psychophysical experiments. They found that a non-linear luminance coding, modeled as a compressive transducer preceding a linear spatial filter, improved the prediction of perceived edge locations but not of perceived bar locations. Their compressive transducer was given by:

$$r(x) = \frac{I}{I + S}, \quad (7)$$

where $r(x)$ is the output of the compressive transducer, $I = L(x)/L_m$ the normalized luminance, and S a constant. We added this compressive transducer to our model calculations, with $S = 0.5$ (following Georgeson & Freeman, 1997). Table 3 shows the results for Criterion 1. A slightly greater σ (1.9 versus 1.1 arcminute) was needed to get an amplitude of compensation of 10 arcsecond for the default experiment. With this σ value, the model with the compressive transducer predicted the effects of modulation depth and line width as accurate as the model without the compressive transducer (Table 3

versus Table 2). However, the model with the compressive transducer grossly deviated in the case of line contrast (bottom of Table 3 versus Fig. 5C) whereas the model without the compressive
 455 transducer agreed perfectly with the experimental results (bottom of Table 2 versus Fig. 5C). This suggests that the non-linear luminance coding used in edge detection plays no role in bar detection.

Table 3 Calculated amplitude of compensation (in arcsecond) as a function of modulation depth, line width, and line contrast for criterion 1 with compressive transducer

Criterion 1 with compressive transducer with $\sigma = 1.9'$	
$m = 0.04$	5
0.08	10
0.16	20
$2\alpha = 0.6'$	20
1.2'	10
2.4'	5
$(L_L - L_m) / L_m = -1.0$	10
-0.5	39
+0.33	-108

460

Edges are supposed to be detected by filters that have a range of sizes. In this way, the visual system is able to address both sharp edges and gradual changes in intensity (Marr & Hildreth, 1980; Georgeson, May, Freeman, & Hesse, 2007; McIlhagga & May, 2012). Interestingly, the filter size (σ) did not

465 depend on the line width in our study. This finding, together with the observation that bar detection
requires a linear filter whereas edge detection requires a non-linear filter, suggests that the detection of
(narrow) bars and edges are different entities in the human visual system.

In conclusion, illusory movement seems to be a logical consequence of the way lines are filtered and
470 localized in the visual system.

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APPENDIX

With $\sin \omega t = 1$ and $\alpha/\sigma \ll 1$, Eq. (1) can be rewritten as:

$$480 \quad L(x) = L_1(x) + L_2(x) + L_3(x), \quad (8)$$

with

$$L_1(x) = L_m,$$

$$L_2(x) = (L_L - L_m) 2\alpha \delta(x-a),$$

and

$$485 \quad L_3(x) = \text{sign}(x-a) m L_m,$$

where δ is the Dirac delta function. Equations (4) and (8) can be combined into:

$$\begin{aligned} g(x) &= g_1(x) + g_2(x) + g_3(x) \\ &= L_1(x) * h(x) + L_2(x) * h(x) + L_3(x) * h(x). \end{aligned} \quad (9)$$

This yields:

$$g_1(x) = L_m, \quad (10a)$$

$$490 \quad g_2(x) = (L_L - L_m) 2\alpha \sqrt{\frac{c}{\pi}} e^{-c(x-a)^2} \approx (L_L - L_m) 2\alpha \sqrt{\frac{c}{\pi}} (1 - c(x-a)^2), \quad (10b)$$

and

$$\begin{aligned} g_3(x) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{2}{iX} e^{-X^2/4c} m L_m e^{iX(x-a)} dX = \frac{2m L_m}{2\pi} \int_{-\infty}^{\infty} \frac{1}{X} e^{-X^2/4c} \sin(X(x-a)) dX \\ &\approx \frac{2m L_m (x-a)}{2\pi} \int_{-\infty}^{\infty} e^{-X^2/4c} dX \\ &= 2m L_m \sqrt{\frac{c}{\pi}} (x-a), \end{aligned} \quad (10c)$$

where $c = 1/(2\sigma^2)$. Finally, requiring

$$\frac{dg}{dx} = 0 \text{ for } x=0 \quad (11)$$

495 results in Eq. (6).

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